Calculation Policy

for

Mathematics

Haydon Bridge Partnership

Revised November 2017

Introduction

The following calculation policy has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the Key Stage 1 and 2 phases. It was revised slightly in November 2017 by the Maths Co-ordinators within the partnership, having experienced working with the new curriculum for several years and also having experiences the new format and level of the KS1 and KS2 SATs. It is expected that all schools in the partnership will follow as closely as possible the methods outlined within the document. This will greatly aid their understanding of different methods and also the transition to High School Maths. Elements of the widely used White Rose Calculation policy have also been added or referred to.

Please note that early learning in number and calculation in Reception follows the 'Development Matters' EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age stage expectations

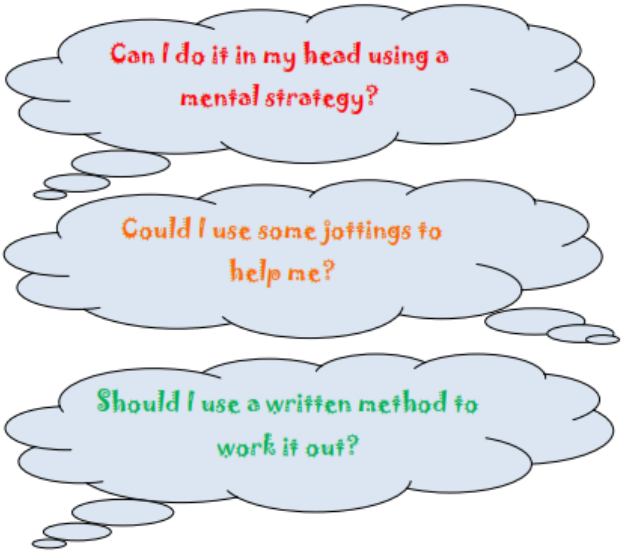
The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014 and the expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons. Children should always have opportunities to move from the **concrete,** through **pictorial** representations to the **abstract** , often referred to as **CPA**

Choosing a calculation method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to do a calculation, to ensure they select the most appropriate method for the numbers involved.



Approximate/Estimate
Calculate
Check

The progression within calculation in the White Rose Hub Calculation policy is as recommended below. It has been included here as an additional resource for staff.

Addition pages have been added into the Calculation Policy with extracts from the WRH policy.

	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.
Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Arrays 2d × 1d using base 10	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first(up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

ADDITION
Year 1

Add one-digit and two-digit numbers to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on. Consider very carefully the language used!

Remember the sequence -

concrete ….. to pictorial ...to abstract…….. Children need loads of concrete experiences!!

See next page…..

Children should:

- Have access to a wide range of counting equipment such as everyday objects, number tracks and number lines, Base 10 apparatus, arrow cards, bead strings etc., and be shown numbers in different contexts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: 8 + 3 = ☐ 5 + ☐ = 8 and 5 + 3 + 1 = ☐

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

8 + 5

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

Key vocabulary:

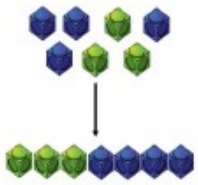
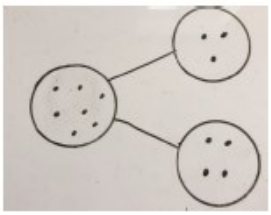
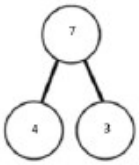
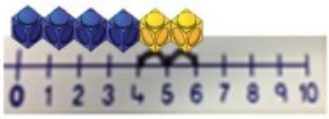
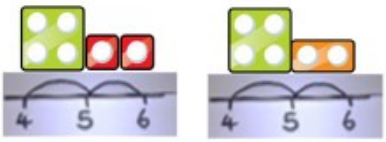
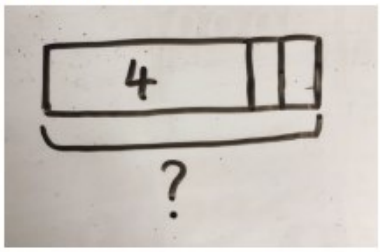
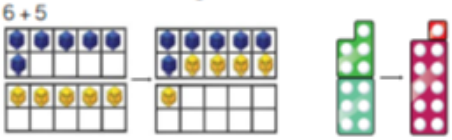
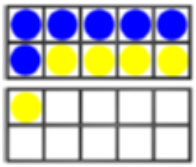
add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Key skills for addition at Y1:

- Count, read and write numbers to 100 in numerals, incl. 1—20 in words
- Represent , recall and use number bonds and related subtraction facts within 20.
- Count to and across 100 boundary , forwards and backwards, beginning with 0 or 1, or from any given number.
- Count in multiples of twos, fives and tens.
- Solve simple, practical 1-step problems involving addition, using objects, number lines and pictorial representations.

ADDITION
Year 1 - continued

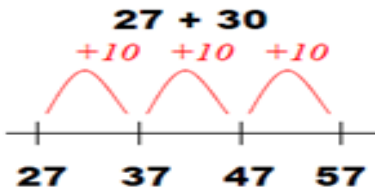
Add one-digit and two-digit numbers to 20

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.  	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 
Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4

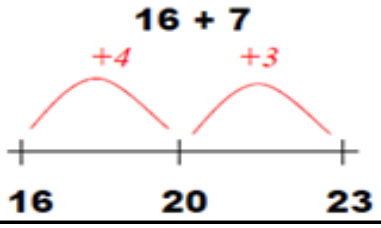
ADDITION
Year 2
Add with two-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

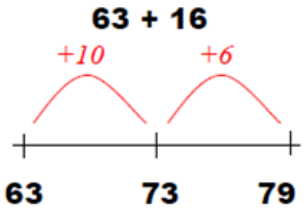
Add 2-digit numbers and tens:



Add 2-digit numbers and units:



Use empty number lines, Base 10 apparatus, hundred squares etc. to build confidence and fluency in mental addition skills.



	6	3
+	1	6
	7	9

	6	3
+	1	9
	8	2
	1	

Model calculations with practical apparatus as much as possible.

It is felt that as the children will have lots of practical experience of adding in columns, going straight to the compact method was best. Some schools use a method called addition alley. It was agreed that this would be an acceptable method to use providing it was phased out and the compact method used by the end of year 3.

Stage 1 - Add 2 digit quantity and a 1 digit quantity - without exchanging any units for tens

Stage 2 - Add 2 digit quantity and a 1 digit quantity **with** exchanging NOTE - Put any amounts exchanged **underneath**.

Stages 3 and 4 - as above but with two 2 digit amounts

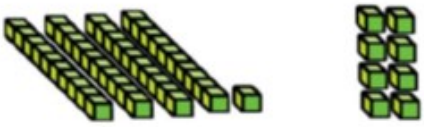
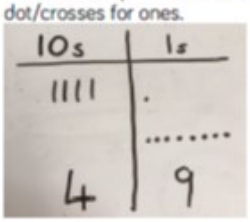
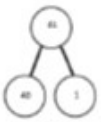
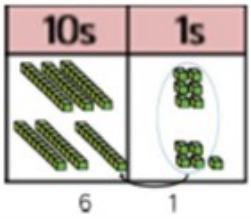
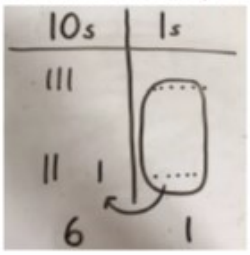
Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Key skills for addition at Y2:

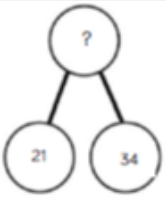
- Add a 2-digit number and ones (e.g. 27 + 6)
- Add a 2-digit number and tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 47)
- Add three single-digit numbers (e.g. 5 + 9 + 7)
- Show that adding can be done in any order (the commutative law).
- Recall and use addition facts to 20 fluently, and derive and use related facts up to 100.
- Count in steps of 2, 3 and 5 and count in tens from any number, forward and backward.
- Recognise the place value of 2-digit numbers (tens and ones)
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.
- Identify, represent and estimate numbers using different representations including the number line.

ADDITION
Year 2
Add with two-digit numbers

Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

Concrete	Pictorial	Abstract
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 4 1 + 8 4 9
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25 	Chidlren to represent the base 10 in a place value chart. 	Looking for ways to make 10. 36 + 25 = 1 5 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: 36 +25 61 1

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

21 + 34 = 55. Prove it

21
+34
___

21 + 34 =
□ = 21 + 34

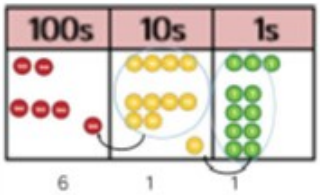
Calculate the sum of twenty-one and thirty-four.

Missing digit problems:

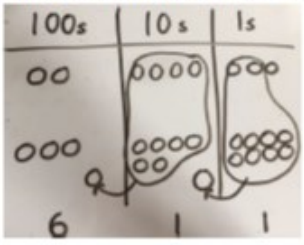
10s	1s
	1
	?
?	5

And then into year 3….

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



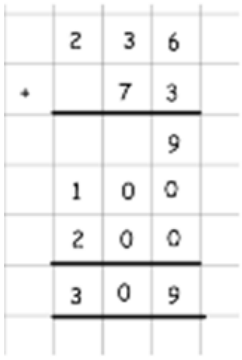
Chidren to represent the counters in a place value chart, circling when they make an exchange.



243
+368
611
1 1

ADDITION
Year 3

Add numbers with up to 3-digits



	2	3	6
+		7	3
			9
	1	0	0
	2	0	0
	3	0	9

Some children may need the **expanded column addition** method first to be able to calculate HTU + TU but most children will be able to go straight to compact method, **particularly with lots of concrete experiences**

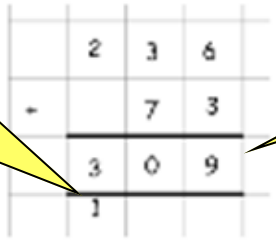
Add the units first in preparation for the compact method.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to understand the value of the digits in the columns and hence the need to work in columns.

Move to the compact **column addition** method, first without, and then with exchanging.

'Carry' numbers underneath the bottom line.



	2	3	6
+		7	3
	3	0	9
	1		

Add the units first.

Remind pupils that the actual value is '**three tens** add **seven tens**', not three add seven', which equals **ten tens**.

Children who are very secure and confident with 3-digit expanded column addition should be moved onto the **compact column addition** method, being introduced to 'exchanging / carrying' in stages. Exchange units, then tens and then both.

Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary, increase, vertical, 'carry', expanded, compact**

Key skills for addition at Y3:

- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, incl. those exceeding 100.
- **Add a three-digit number and ones mentally (175 + 8)**
- **Add a three-digit number and tens mentally (249 + 50)**
- **Add a three-digit number and hundreds mentally (381 + 400)**
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.
- Count from 0 in multiples of 4, 8, 50. and 100.
- Compare and order numbers up to 1000.
- Find 10 or 100 more or less than a given number.

ADDITION
Year 4

Page 1 - Add numbers with up to 4 digits

The compact column method should be well established by now. Extend to 3 digit numbers and greater by always **adding units first**, and 'carrying' numbers **underneath** the calculation. Also include money and measures contexts. Discontinue referring to 'addition alley'.

e.g. 3517 + 396 = 3913

	3	5	1	7
+		3	9	6
	3	9	1	3
		1	1	

'Carry' numbers **underneath** the bottom line

Add units first

To begin with, reinforce correct place value by reminding them of actual values:

- **7 + 6 = 13. Put 3 in the units column and carry the 1 ten to the tens column.**
- **1 tens + 9 tens + 1 ten = 11 tens. Put 1 ten in the tens column and carry 1 hundred to the hundreds column.**
- **5 hundreds + 3 hundreds + 1 hundred = 9 hundreds etc.**

Eventually, simplify language using "of these"

(The significance of the phrase "of these" becomes important and more obvious as the children start longer short divisions! - See later)

What not to say:

7 + 6 = 13. Put 3 down and 1 on the doorstep.

Extend to adding 3 whole numbers e.g. 3517+ 396 + 43

Use and apply this method to money and measurement values.

ADDITION
Year 4

Page 2 - Add numbers with up to 4 digits

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, **thousands, hundreds, digits, inverse**

Key skills for addition at Y4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, i.e. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

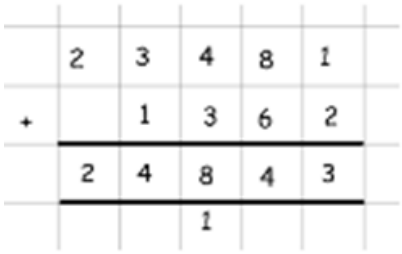
ADDITION
Year 5

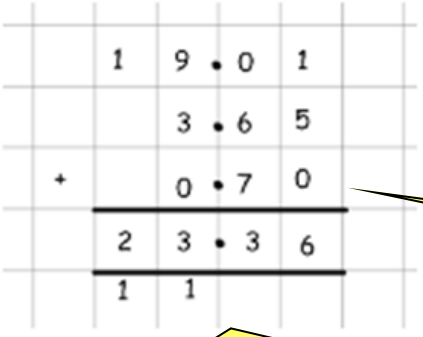
Add numbers with more than 4 digits

including money, measures and decimals with different numbers of decimal places.

The decimal point should be aligned in the same way as the other place value columns. Decimal points should be **on the dividing line** between the units and the tenths, not in a column of their own!

Numbers should exceed 4 digits.





Pupils should be able to add more than two values, carefully aligning place value columns.

Empty decimal places can be filled with zero to show the place value in each column.

Say '6 tenths add 7 tenths' to reinforce place value. Eventually, use the phrase "of these" to simplify language.

Children should understand the place value of **tenths and hundredths** and use this to align numbers with different numbers of decimal places.

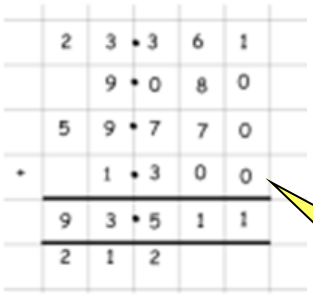
Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse & **decimal places, decimal point, tenths, hundredths, thousandths**

Key skills for addition at Y5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies i.e. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

ADDITION
Year 6

Add several numbers of increasing complexity

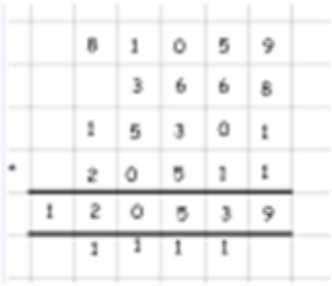


Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

- Zeros could be added into any empty decimal places, to show there is no value to add.

Empty decimal places should be filled with zero to show the place value in each column.



Adding several numbers with more than 4 digits.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Y6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

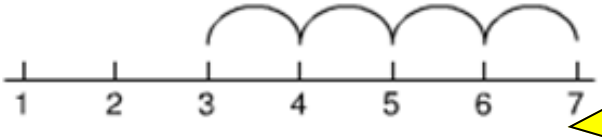
SUBTRACTION Year 1

Subtract from numbers up to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below. See overleaf for advice from White Rose Hub Calculation Policy.

Subtract by taking away

Count back in ones on a numbered number line to take away, with numbers up to 20



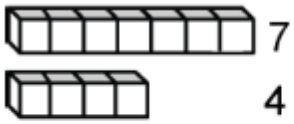
$7 - 4 = 3$

Remember always to reinforce the link between addition and subtraction.

Model subtraction using hundred squares and numbered number lines/ tracks and practically. - See next page

Find the 'difference between'

This will be introduced practically with the language **'find the difference between'** and '**how many more?**' in a range of familiar contexts.



'Seven is 3 more than four'

'I am 2 years older than my sister'

Mental subtraction

Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

Key skills for subtraction at Y1:

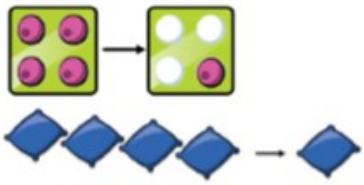
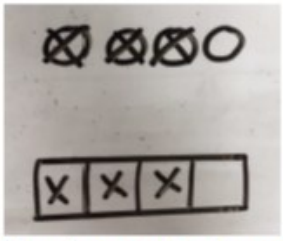
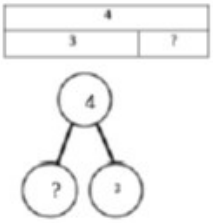
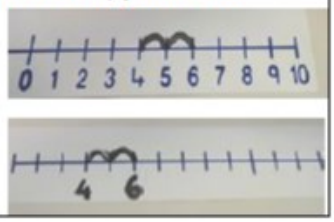
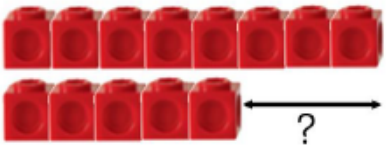
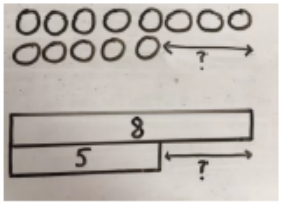
- Given a number, say **one more or one less.**
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (i.e. bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.

SUBTRACTION
Year 1

Subtract from numbers up to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below. See below advice rom White Rose Hub Calculation Policy.

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 - 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = □ = 4 - 3 
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 - 2 = 4 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 - 5, the difference is □ Children to explore why 9 - 6 = 8 - 5 = 7 - 4 have the same difference.

SUBTRACTION
Year 2
Subtract with 2 digit numbers

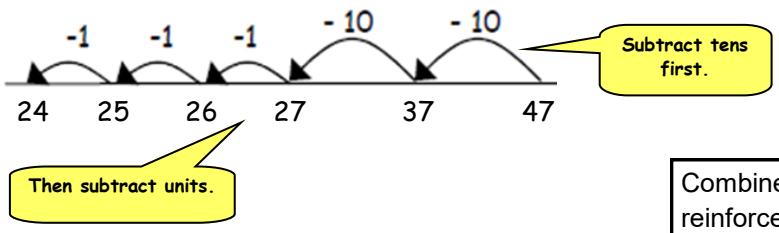
Subtract on a number line by counting back, aiming to develop mental subtraction skills. The strategy should be modelled using Base 10 apparatus etc.

This strategy will be used for:

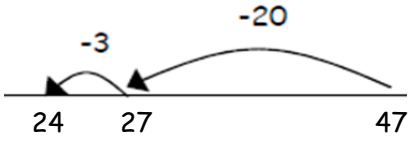
2-digit numbers subtract units (by taking away / counting back) e.g. 36—7
2-digit numbers subtract tens (by taking away / counting back) e.g. 48—30
Subtracting pairs of 2-digit numbers (see below:)

Subtracting pairs of 2-digit numbers on a number line:

47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:



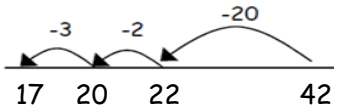
Move towards more efficient jumps back, as below:



Combine methods with use of a hundred square to reinforce understanding of number value and order.

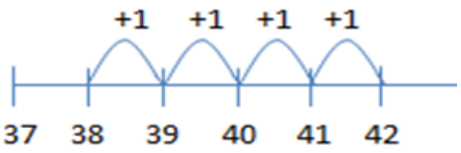
Teaching children to **bridge through ten** can help them to become more efficient,

for example 42—25:



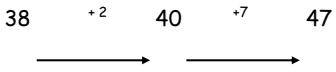
Mental strategy - subtract numbers close together by **counting on:**

42 − 38 = 4



Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

They should also be encouraged to use more efficient ways to count on than counting in ones, using number bonds, e.g.

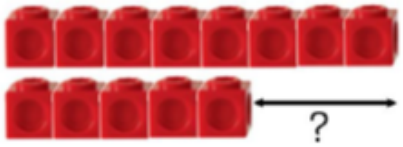
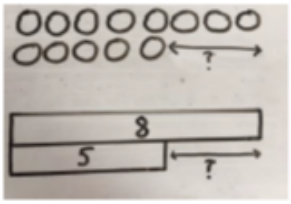
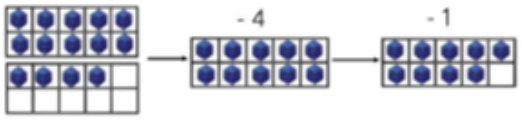
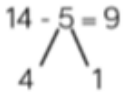
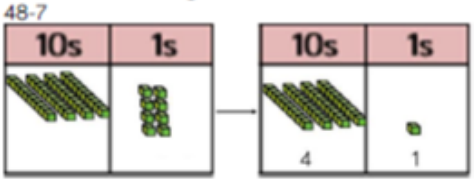
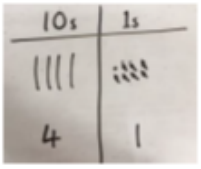
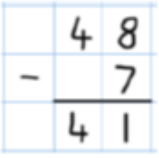
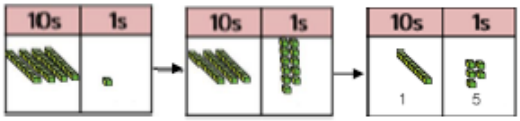
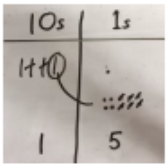
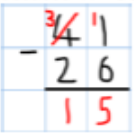


Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? **difference, count on, strategy, partition, tens, units**

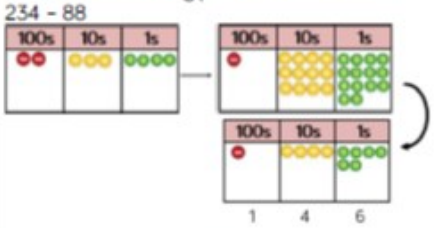
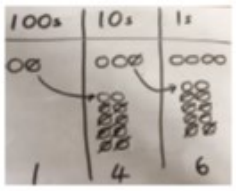
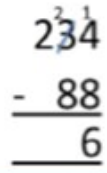
Key skills for subtraction at Y2:

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

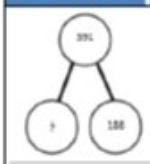
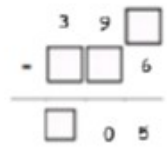
SUBTRACTION
Year 2
Subtract with 2 digit numbers

Concrete	Pictorial	Abstract
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is ☐ Children to explore why 9 - 6 = 8 - 5 = 7 - 4 have the same difference.
Making 10 using ten frames. 14 – 5 	Children to present the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend.  14 – 4 = 10 10 – 1 = 9
Column method using base 10. 48-7 	Children to represent the base 10 pictorially. 	Column method or children could count back 7. 
Column method using base 10 and having to exchange. 41 – 26 	Represent the base 10 pictorially, remembering to show the exchange. 	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 

And then into Year 3...

Column method using place value counters. 234 – 88 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal colum method. Children must understand what has happened when they have crossed out digits. 

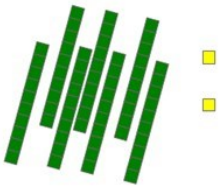
Conceptual variation; different ways to ask children to solve 391 - 186

	Raj spent £391, Timmy spent £186. How much more did Raj spend? Calculate the difference between 391 and 186.	☐ = 391 – 186 391 -186 ___ What is 186 less than 391?	Missing digit calculations 

SUBTRACTION
Year 3
Subtract with 2 and 3 digit numbers

STEP 1: introduce 'exchanging' through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

72 - 47



Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

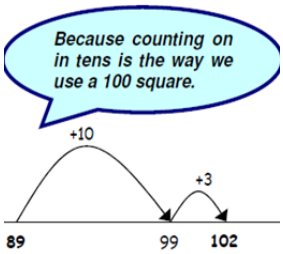
Column subtraction method

STEP 2: Once pupils are secure with the understanding of 'exchanging', they can use the column method to subtract any 2 then 3-digit numbers.

	$^{6}\cancel{7}$	$^{1}2$
-	4	7
	2	5

	$^{2}\cancel{3}$	$^{1}4$	6
-	1	8	2
	1	6	4

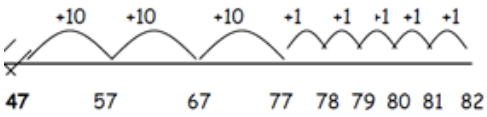
Use vocabulary 'change', not 'borrow'.



Counting on as a mental strategy for subtraction:

Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121—118), and also for numbers that are "nearly" multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 131—79, or calculating change from £1 etc.).

Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:



Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units **exchange, decrease, hundreds, value, digit**

Key skills for subtraction at Y3:

- Subtract mentally a: **3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds** .
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number.
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10
- Read and write numbers up to 1000 in numerals and words.
- Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

SUBTRACTION Year 4

Subtract with up to 4 digit numbers

Column subtraction method using larger numbers

As introduced in Y3, but moving towards larger numbers and money.

Language can be simplified using the phrase "of these".....

Change one of these ....

....into ten of these.

	2	$^{6}\cancel{7}$	$^{1}5$	4
-	1	5	6	2
	1	1	9	2

	£		
	$^{4}\cancel{5}$.	$^{1}1$	7
-	2 .	4	3
	2 .	7	4

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved— mental, counting on, counting back or written method. Counting on is particularly useful for finding the difference.

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, **inverse**

Key skills for subtraction at Y4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal place.
- Find 1000 more or less than a given number. Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000.
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

SUBTRACTION Year 5

Subtract with at least 4 digit numbers

including money, measures, decimals.

Compact column subtraction

Subtracting with larger integers.

	$^{2}\cancel{3}$	$^{10}\cancel{1}$	$^{1}0$	$^{3}\cancel{4}$	$^{1}6$
-		2	1	2	8
	2	8	9	1	8

Including calculations which need several changes.

418.5 - 78.26

↓

	$^{3}\cancel{4}$	$^{1}1$	8 .	$^{4}\cancel{5}$	$^{1}0$
-		7	8 .	2	6
	3	4	0 .	2	4

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Place a 'zero' in any empty places to aid understanding of what to subtract in that column.

Create lots of opportunities for subtracting and finding differences with money and measures.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, **tenths, hundredths, decimal point, decimal**

Key skills for subtraction at Y5:

- Subtract numbers mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

SUBTRACTION
Year 6

Subtract with increasingly large and more complex numbers and decimal values

	6	$^{7}\cancel{8}$	$^{1}\cancel{1}^{9}0$	$^{1}0$	$^{3}\cancel{4}$	$^{1}5$
-	4	2	3	1	3	8
	2	5	6	9	0	7

Using the compact column method to subtract more complex integers. Note the effect of the two zeros in the middle two columns.

5268 - 734.76

↓

	$^{4}\cancel{5}$	$^{1}2$	6	$^{7}\cancel{8}$. $^{10}\cancel{0}^{9}$		$^{1}0$
-		7	3	4 .	7	6
	4		3	3 .	2	4

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Empty places should be filled with **zeros** to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Key skills for subtraction at Y6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit
- Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

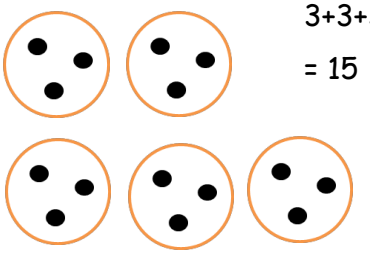
MULTIPLICATION
Year 1

Multiply with concrete objects, arrays and pictorial representations

How many legs will 3 teddies have?

There are 3 sweets in one bag.

How many sweets are in 5 bags altogether?



3+3+3+3+3

= 15

Give children experience of counting equal group of objects in 2s, 5s and 10s.

Present practical problem solving activities involving counting equal sets or groups, as above.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key skills for multiplication at Y1:

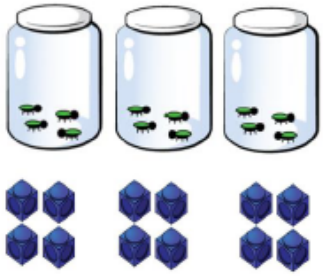
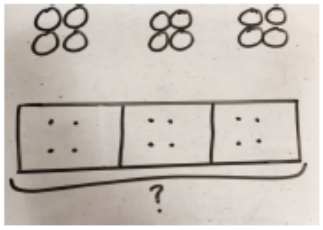
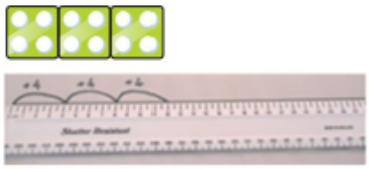
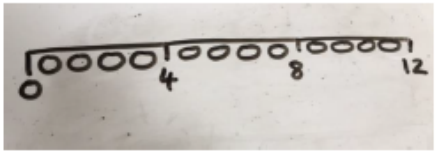
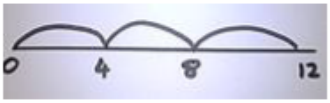
- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.

MULTIPLICATION
Year 1

Multiply with concrete objects, arrays and pictorial representations

These skills will be visited in Year 1 and Year 2

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4  Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. 3 × 4 = 12 

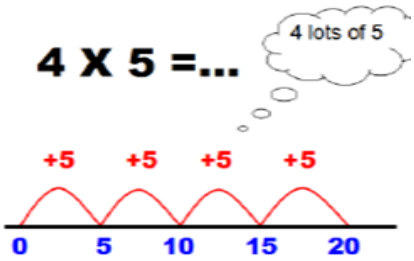
MULTIPLICATION
Year 2

Multiply using arrays and repeated addition

(using at least 2s, 5s and 10s)

Use repeated addition on a number line:

Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.



Use arrays:

Constantly reinforce the link between multiplication and division.

3 x 5 = 15

5 x 3 = 15

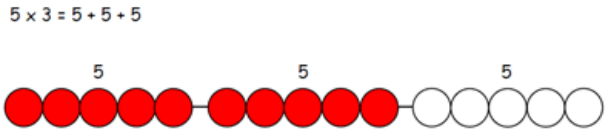
4 X 5 = 20

3 x 5 = 5 + 5 + 5 = **15**

5 x 3 = 3 + 3 + 3 + 3 = **15**

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as **3 x __ = 15**

Use practical apparatus:



Use mental recall:

Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practice in counting and understanding of the operation.

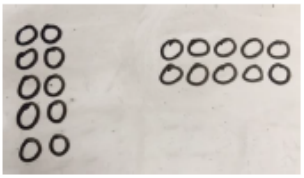
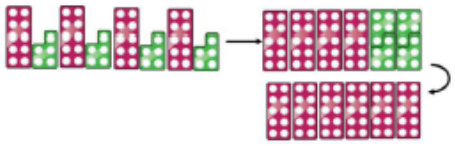
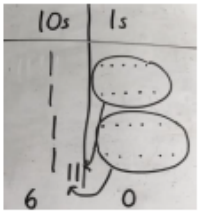
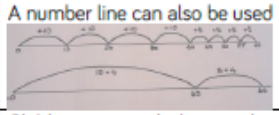
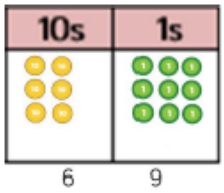
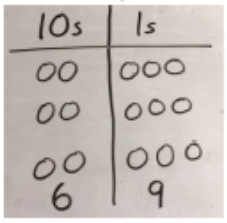
Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...

Key skills for multiplication at Y2:

- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements **using the x and = signs**.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

MULTIPLICATION
Year 2

Multiply using arrays and repeated addition

Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 x 15 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3×23 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3×23 $20 \quad 3$ $3 \times 20 = 60$ $3 \times 3 = 9$ $60 + 9 = 69$ $23 \times 3 = 69$

MULTIPLICATION
Year 3

Multiply 2-digits by a single digit number

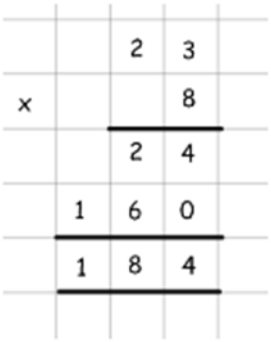
In some schools in the partnership, children are introduced to the grid method for multiplying 2-digit by single-digits:

Eg. 23 x 8 = 184

x	8
3	2 4
2 0	1 6 0

Line up the digits in place value columns.

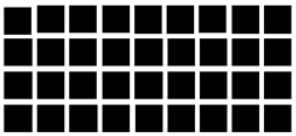
Then add in columns: 1 8 4 (ensure the 8 isn't added in as well)



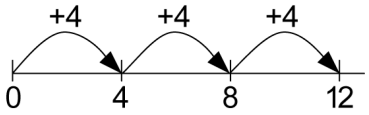
Introduce the grid method with children by physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format.

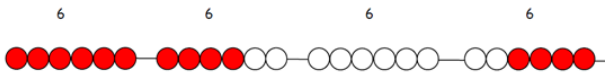
To do this, children must be able to:

- Partition numbers into tens and units.
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value.
- Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:



9 x 4 = 36





Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., **partition, grid method, multiple, product, tens, units, value**

Key skills for multiplication:

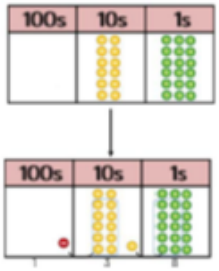
- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including **2-digit x single-digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g. using commutativity

(**4 × 12 × 5** = 4 × 5 × 12 **= 20 × 12** = 240)

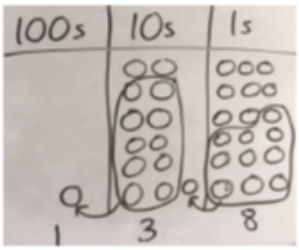
and for missing number problems ☐ x 5 = 20, 3 x ☐ = 18, ☐ x ☐ = 32

MULTIPLICATION
Year 3
Multiply 2-digits by a single digit number

Formal column method with place value counters.
6 x 23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline 1\ 1 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6 × 23

23	23	23	23	23	23

?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

6 × 23 =

□ = 6 × 23

$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \\ \hline \end{array}$$

What is the calculation?
What is the product?

100s	10s	1s

MULTIPLICATION
Year 4

Multiply 2 and 3-digits by a single digit

Using all multiplication tables up to **12 x 12**

Some children will be developing their grid method skills but most will more onto more formal methods

Eg. 136 x 5

X	5
6	3 0
3 0	1 5 0
1 0 0	5 0 0

Line up the digits in place value columns.

Encourage estimating and checking to ensure accuracy.

Then add in columns: 6 8 0

Developing the expanded column method:

Many children in Year 4, however, are ready to move onto **short multiplication** (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, **and** are already confident in "carrying" for written addition.

When solving problems this way, children should be able to:

- **Approximate before they calculate**, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer.

  e.g: **—346 x 9** is approximately 350 x 10 = **3500**

- Record an approximation to check the final answer against.
- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.
- Recall all times tables **up to 12 x 12**

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times, as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Key skills for multiplication at Y4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12.**
- Recognise place value of digits in up to 4-digit numbers.
- Use place value, known facts and derived facts to multiply mentally.
- Use commutativity and other strategies mentally 3 x 6 = 6 x 3 , 2 x 6 x 5 = 10 x 6 , 39 x 7 = 30 x 7 + 9 x 7.
- Solve problems with increasingly complex multiplication in a range of contexts
- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

MULTIPLICATION
Year 5

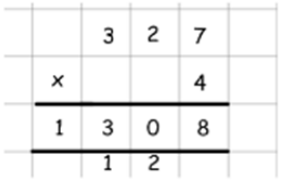
Multiply up to 4-digits by 1 or 2 digits

Please Note

When discussed at the Haydon Bridge Partnership Maths co-ordinators meeting, we decided that extending expanded column multiplication to TU x TU and beyond is not to be recommended and the grid method should now be dropped.

Introducing column multiplication

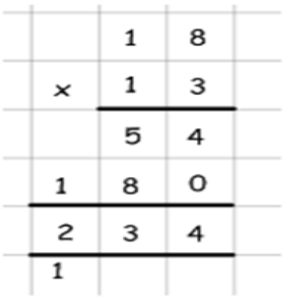
Short multiplication for multiplying by a single digit



Pupils could be asked to work out a given calculation using the grid method initially, and then compare it to the column method. What are the similarities and differences?

Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits



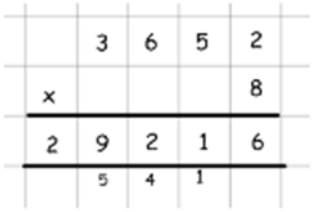
18 x 3 on the 1st row

(8 x 3 = 24, carrying the 2 for twenty, then 1 ten x 3).

18 x 10 on the 2nd row.

Remind the children that they are now multiplying by a multiple of ten so they should put the place holder zero in units first, then say 8 x 1, and 1 x 1.

Moving towards more complex numbers:



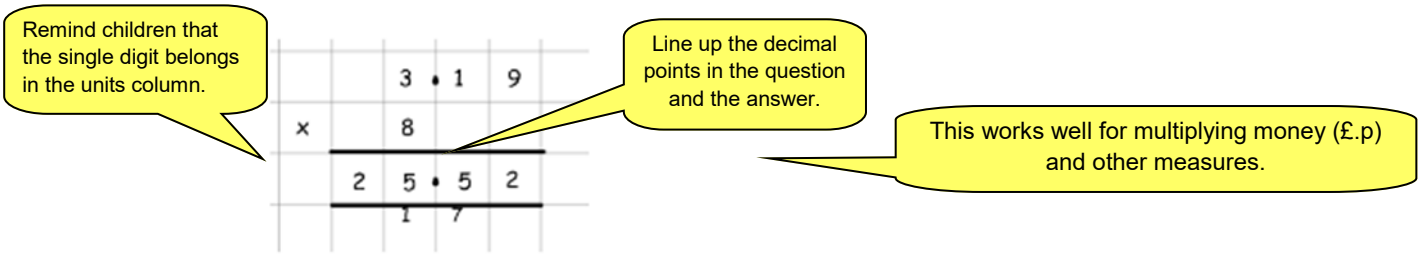
Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, 'carry'**

Key skills for multiplication at Y5:

- Identify multiples and factors, using knowledge of **multiplication tables to 12 x 12.**
- Solve problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

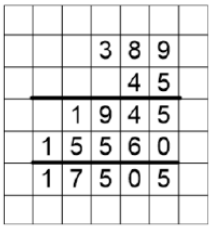
MULTIPLICATION
Year 6

Short and long multiplication as in Y5, and multiply decimals with up to 2 d.p by a single digit



Children working at greater depth will be expected to calculate quantities with 2 decimal places by quantities with 1 decimal place.

In these cases, teach the children to multiply the numbers by e.g. 100 and also 10 first and then divide their answer accordingly.



E.g To calculate 3.85 x 4.5

1. Multiply 3.85 x 100 and 4.5 x 10
2. Carry out the calculation as normal
3. Then, as your answer is 1000 times too big (100 x 10), divide the answer by 1000. In this case 17.505

When using either method, **the key skill is to <u>estimate first to see what the likely answer will be. This is a really important skill and one that is often not taught! DON'T FORGET!!</u>**

In question 1 - 3 x 8 is about 24 **In question 2** 4 x 4 is about 16

Children will need to be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit**; to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit.**
- Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number.**

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, „carry", **tenths, hundredths, decimal**

Key skills for multiplication at Y6:

- Recall multiplication facts for all times tables up to **12 x 12 (as Y4 and Y5).**
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine levels of accuracy.
- Round any integer to a required degree of accuracy.

DIVISION
Year 1
Group and share small quantities

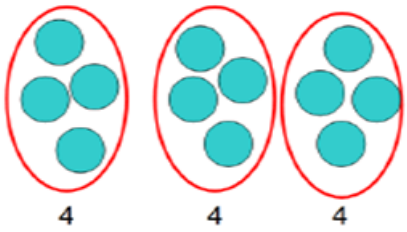
Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**.

How many groups of 4 can be made with 12 stars?

Grouping:

Sharing:

These are given out 1 by 1



12 shared between 3 is 4

Always try and give a division problem in a familiar context at this stage:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement… ?

"18 shared between 6 people gives you 3 each."

Pupils should :

- Use lots of practical apparatus, arrays and picture representations
- Be taught to understand the difference between „grouping" objects (How many groups of 2 can you make?) and "sharing" (Share these sweets between 2 people)
- Be able to count in multiples of 2s, 5s and 10s.
- Find **half** of a group of objects by sharing into 2 equal groups.

Key Vocabulary: **share, share equally, one each, two each…, group, groups of, lots of, array**

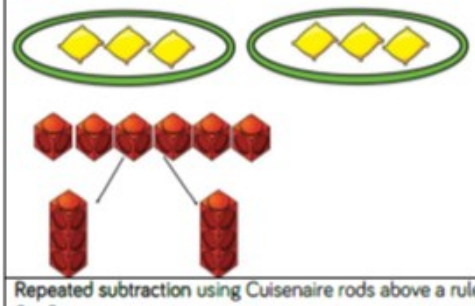
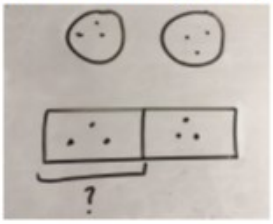
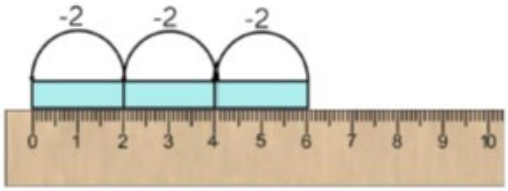
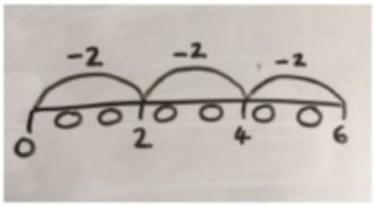
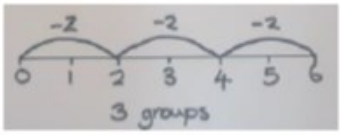
Key number skills needed for division at Y1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

DIVISION
Year 1

Group and share small quantities

Key language: share, group, divide, divided by, half.

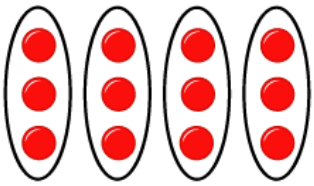
Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

DIVISION
YEAR 2

Group and share small quantities

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:



12 ÷ 3 = 4

This represents **12 ÷ 3, posed as**

How many groups of 3 are in 12?

Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

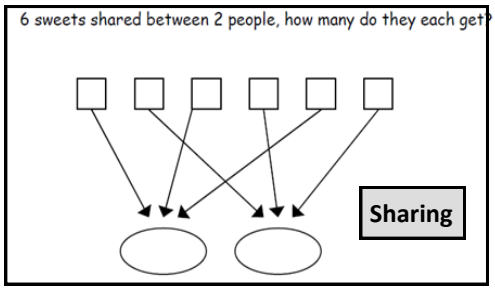
Important!

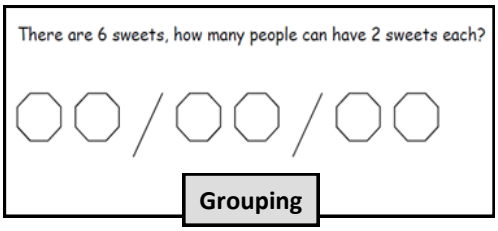
The words "divide" and "divide by" and the symbol "**÷**" are ambiguous.

For example, 28 ÷ 7 can mean two different things.

Note also that objects are "**shared between**", not "shared by".

Know and understand sharing and grouping:





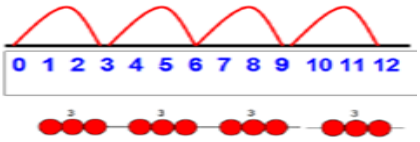
Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out "How many groups of ____ in ____ ?"

Pupils could use a bead string or practical apparatus to work out problems like "A CD costs £3. How many CDs can I buy with £12?"

This is an important method to develop understanding of division as grouping.



12 ÷ 3 = 4

Pose **12 ÷ 3** as
'How many groups of 3 are in 12?'

When sharing, make connection between 12 shared between 3 and $\frac{1}{3}$ of 12.

Key Vocabulary: **share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, shared between**

Cuisenaire and thinking blocks are useful resources for division.

Key number skills needed for division at Y2

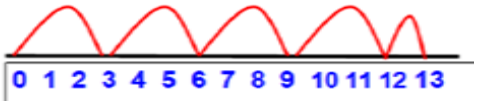
- Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

DIVISION
Year 3
Divide 2-digit numbers by a single digit

(where there is no remainder in the final answer)

Grouping on a number line:

13 ÷ 3 =



STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of **remainders,** as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for 'carrying' remainders across within the short division method.

Short division:

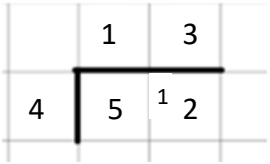
Limit numbers to **NO** remainders in the answer **OR** carried (each digit must be a multiple of the divisor).

STEP 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all.

Note: The language used at this point is crucial to understanding. It is meaningless to say "How many threes in 9?" It is much better to say "If you share 9 tens between 3 people, how many do they get each?" (They get 3 tens each so write 3 in the tens column above the 9). This format may seem wordy by comparison, but it can later be shortened to: "Share 9 of these between 3 people." The phrase "of these" can be used in any column.

Short division:

Limit numbers to **NO** remainders in the final answer, but with remainders occurring within the calculation.



STEP 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 96 ÷ 4), and be taught to 'carry' the remainder onto the next digit, still using appropriate language, e.g. "Change 1 of these into 10 of these."

If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.

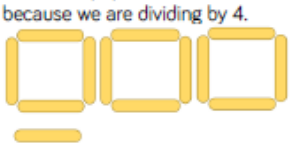
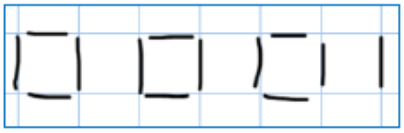
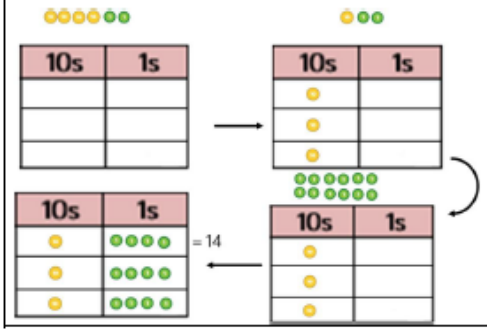
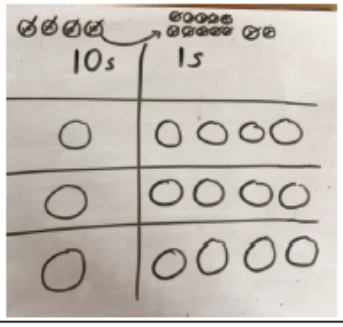
Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, shared between, **inverse, short division, 'carry', remainder, multiple**

Key number skills needed for division at Y3:

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, Connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

DIVISION
Year 3

Divide 2-digit numbers by a single digit

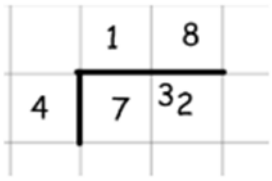
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

DIVISION
Year 4

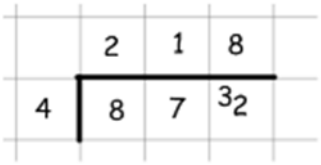
Divide up to 3-digit numbers by a single digit

Without remainders initially and then need to emphasise that the remainder can be expressed as a fraction

Continue to develop short division:



STEP 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (**those that do not result in a final remainder**—see steps and appropriate language in Y3), but must understand how to calculate remainders, using this to 'carry' remainders within the calculation process (see example).



STEP 2: Pupils move onto dividing numbers with up to **3-digits** by a single digit. However, problems and calculations provided should **not result in a final answer with remainder** at this stage**.** Children who exceed this expectation may progress to Year 5 level.

When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always 'carry' the unused quantity over to the next digit as a remainder.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, **divisible by, factor**

Key number skills needed for division at Y4:

- Recall multiplication and division facts for all numbers up to 12 x 12.
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number.
- Pupils practise mental methods and extend this to three-digit numbers to derive facts,

  for example 200 × 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

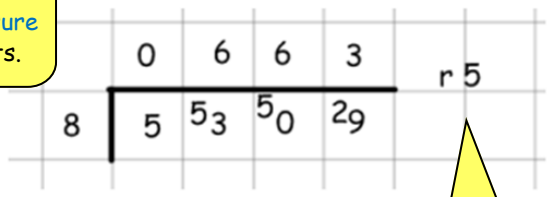
DIVISION
Year 5

Divide up to 4 digits by a single digit

Including those with remainders expressed as remainders, fractions or (later on) decimals.

Short division, including remainder answers:

Include money and measure contexts.



See Y6 for how to continue the short division to give a decimal answer for children who are confident.

Children need to understand that this remainder can also be expressed as e.g. 5/8ths

Short division with remainders:

Now that pupils are introduced to examples that give rise to remainder answers, **pupils need to consider the meaning of any remainder** depending upon the context of the problem. Examples:

1) Share 25 marbles between 4 children: 25 ÷ 4 = 6 r 1

2) 25 people are asked to make groups of 4. How many groups of 4 can be made? 25 ÷ 4 = 6 r 1 ⟶ 6 groups

3) 25 people need to be taken by taxi to a theatre. Each taxi holds 4 passengers. How many taxis are needed?

25 ÷ 4 = 6 full taxis, remainder 1 person ⟶ 7 taxis.

This is the process of actually understanding what the remainder means. Appropriate rounding might be needed to answer a question after the calculation has been done.

If children are confident and accurate:

Introduce **long division** for pupils who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Year 6 expectation - see Year 6.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, **quotient, prime number, prime factors, composite number (non-prime)**

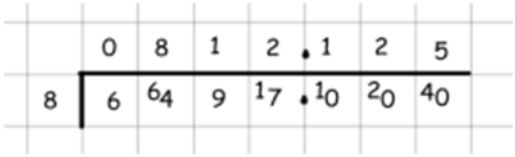
Key number skills needed for division at Y5:

- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g.98÷4=24r2=241/2= 24.5 ˜ 25).
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

DIVISION
Year 6
Divide at least 4 digits by both single-digit and 2-digit numbers

(including decimal numbers and quantities)

Short division, for dividing by a single digit: e.g. 6497 ÷ 8



Short division with remainders:

Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers.

Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder:

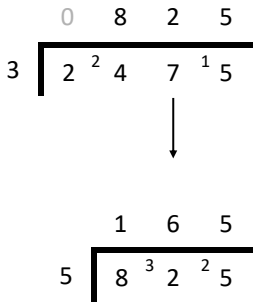
In this example, rather than expressing the remainder as **r 1**, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

E.g. Divide £25 between 4 people: 25 ÷ 4 ⟶ 25.00 ÷ 4 = 6.25 ⟶ £6.25

Finally…….Dividing by 2 digit numbers!

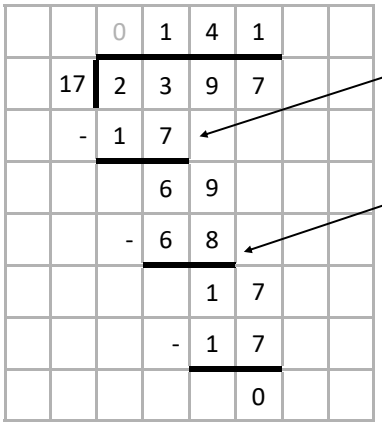
Method 1: Where the divisor can be factorised

Consider the calculation 2475 ÷ 15. Now consider the divisor as 3 x 5. Therefore the calculation can be thought of as 2475 divided by 3 and then by 5.



Method 2: Where the divisor can't be factorised

Consider the calculation 2227 ÷ 17. The divisor cannot be factorised (other than 1 x 17), so there is no alternative but to divide by 17.



1 x 17 = 17. Subtract from 23 to give 6. Then bring down the 9 to join the 6.

4 x 17 = 68 Subtract from 69 to give 1. Then bring down the 7 to join the 1, etc.

Note that the above language does not reinforce the proper values of the numbers, but that there is little alternative!

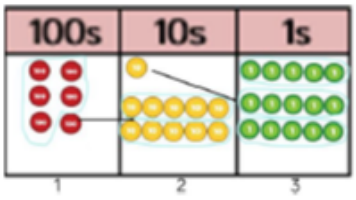
Key Vocabulary: As previously, & common factor
Key number skills needed for division at Y6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret **remainders as whole number remainders, fractions, or by rounding,** as appropriate for the context.
- Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.

DIVISION
Year 6

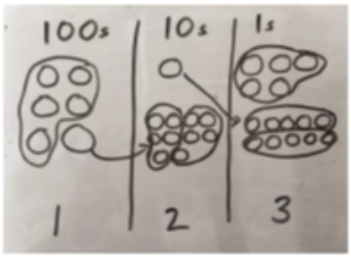
Divide at least 4 digits by both single-digit and 2-digit numbers

Short division using place value counters to group.
615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

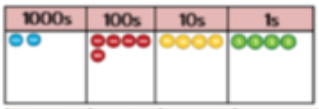
Represent the place value counters pictorially.



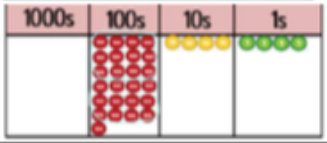
Children to the calculation using the short division scaffold.

$$5\overline{)615}\quad 123$$

Long division using place value counters
2544 ÷ 12

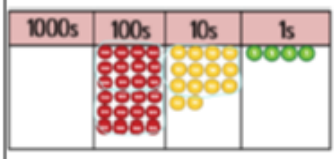


We can't group 2 thousands into groups of 12 so will exchange them.



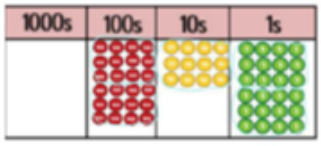
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$12\overline{)2544}\quad 02,\ 24,\ 1$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$12\overline{)2544}\quad 021,\ 24,\ 14,\ 12,\ 2$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$12\overline{)2544}\quad 0212,\ 24,\ 14,\ 12,\ 24,\ 24,\ 0$$

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?

I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5\overline{)615}$$

615 ÷ 5 =

☐ = 615 ÷ 5

What is the calculation?
What is the answer?